Example 1-7

Figure 1-9 shows the one-line diagram of simple three-bus power system with generation at bus 1. The magnitude of voltage at bus 1 is adjusted to 1.05 pu. The scheduled loads at bus 2 and 3 are marked on the diagram. Line impedances are marked in per unit on a 100 MVA base and the line charging susceptances are neglected.

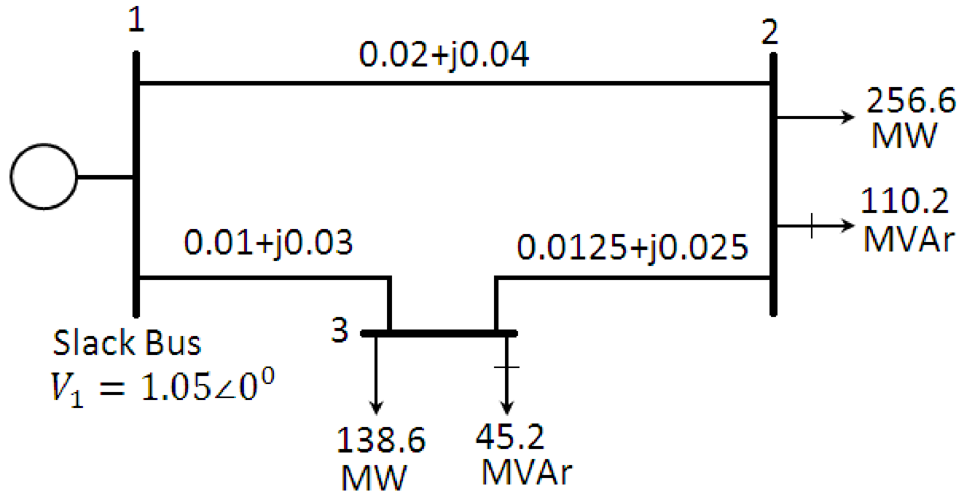

Figure (1-9) One-line diagram of Example 1-7 **(impedances in pu on 100 MVA base)**

a) Using the Gauss-Seidel method, determine the phasorvalue of the voltage at the load buses 2 and 3 (P-Q buses) accurate to four decimal places.
b) Find the slack bus real and reactive power.
c) Determine the line flows and line losses. Construct a power flow diagram showing the direction of line flow.

(a) Line impedances are converted to admittances

$$y_{12} = \frac{1}{0.02 + j0.04} = 10 - j20$$

Similarly, $y_{13} = 10 - j30$, $y_{23} = 16 - j32$. The admittances are on the network shown in Figure 1-10.

At the P-Q buses, the complex lodes expressed in per units are

$$S_2^{sch} = -\frac{(256.6 + j110)}{100} = -2.566 - j1.102\ pu$$

$$S_3^{sch} = -\frac{(138.6 + j45.2)}{100} = -1.386 - j0.452\, pu$$

Since the actual admittances are readily available in Figure 1-10, for hand calculation, we use (1-28). Bus 1 is taken as reference bus (slack). Starting from an initial estimate of

$$V_2^{(0)} = 1.0 + j0.0 \; and \;\; V_3^{(0)} = 1.0 + j0.0,$$

V_2 and V_3 are computed from (1-28) as follows

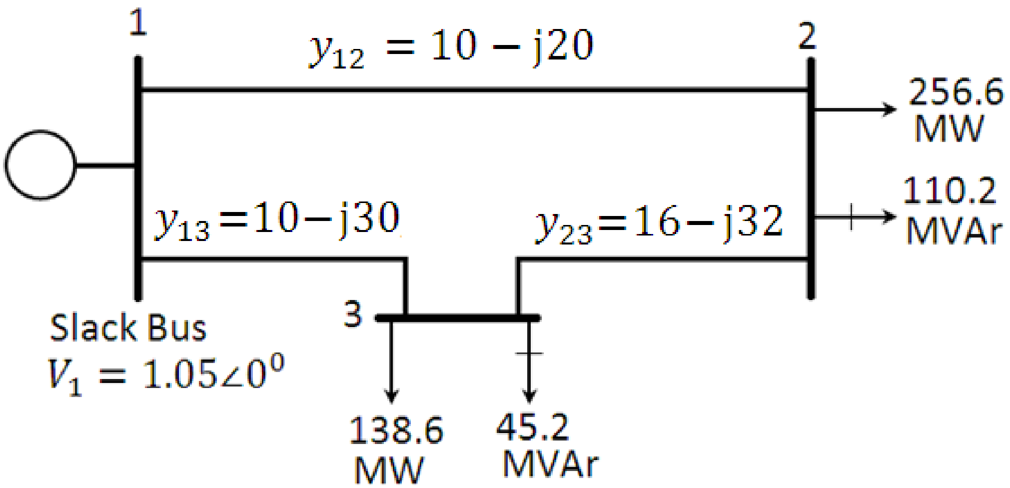

Figure (1-10) One-line diagram of Example 1-7 **(admittances in pu on 100 MVA base)**

$$V_2^{(1)} = \frac{\frac{P_2^{sch} - jQ_2^{sch}}{V_2^{*(0)}} + y_{12}V_1 + y_{23}V_3^{(0)}}{y_{12} + y_{23}}$$

$$V_2^{(1)} = \frac{\frac{-2.566 + j1.102}{1.0 - j0.0} + (10 - j20)(1.05 + j0.0) + (16 - j32)(1.0 + \mathrm{j}0.0)}{(26 - \mathrm{j}52)}$$

$V_2^{(1)}$=0.9825-j0.0310

and

$$V_3^{(1)} = \frac{\frac{P_3^{sch} - jQ_3^{sch}}{V_3^{*(0)}} + y_{13}V_1 + y_{23}V_2^{(1)}}{y_{13} + y_{23}}$$

$$V_3^{(1)} = \frac{\frac{-1.386 + j0.452}{1.0 - j0.0} + (10 - j30)(1.05 + j0.0) + (16 - j32)(0.9825 - \mathrm{j}0.0310)}{(26 - \mathrm{j}62)}$$

$V_3^{(1)}$=1.0011-j0.0353

For the second iteration we have

$$V_2^{(2)} = \frac{\frac{-2.566+j1.102}{0.9825+j0.0310} + (10-j20)(1.05+j0.0) + (16-j32)(1.001+\mathrm{j}0.0353)}{(26-\mathrm{j}52)}$$

$V_2^{(2)}$=0.9816-j0.0520

and

$$V_3^{(2)} = \frac{\frac{-1.386+j0.452}{1.0011-j0.0353} + (10-j30)(1.05+j0.0) + (16-j32)(0.9816-\mathrm{j}0.052)}{(26-\mathrm{j}62)}$$

$V_3^{(2)}$=1.0008-j0.0459

The process is continued and a solution in converted with an accuracy of 5$\times 10^{-5}$ per unit as given below.

$V_2^{(3)}$=0.9808-j0.0578, $V_3^{(3)}$=1.0004-j0.0488
$V_2^{(4)}$=0.9803-j0.0594 $V_3^{(4)}$ =1.0002-j0.0497
$V_2^{(5)}$=0.9801-j0.0598, $V_3^{(5)}$=1.0001-j0.0499
$V_2^{(6)}$=0.9801-j0.0599, $V_3^{(6)}$=1.0000-j0.0500
$V_2^{(7)}$=0.9800-j0.0600, $V_3^{(7)}$=1.0000-j0.0500

The final solution is

V_2=0.9800-j0.0600= 0.98183$\llcorner -3.5035^0$ pu

V_3=1.0000-j0.0500= 1.001125$\llcorner -2.8624^0$ pu

(b) with the knowledge of all bus voltages, the slack bus powers is obtained from (1-27)

$$P_1 - jQ_1 = V_1^*[V_1(y_{12}+y_{13}) - (y_{12}V_2 + y_{13}V_3)]$$

$$P_1 - jQ_1 = 1.05[1.05(20-\mathrm{j}50) - (10-\mathrm{j}20)(0.98-\mathrm{j}0.06) - (10-\mathrm{j}30)(1.0-\mathrm{j}0.05)]$$

$$P_1 - jQ_1 = 4.095 - j1.98 \text{ pu}$$

or the slack bus real and reactive powers are

$$P_1 = 4.095\, pu = 409.5 \text{ MW}$$
$$Q_1 = 1.98\ pu = 189 \text{ MVAr}$$

(b) To find the line flows, first the line current are computed. With line charging capacitors neglected, the line current are

$I_{12} = y_{12}(V_1 - V_2) = (10 - \mathrm{j}20)[(1.05 + \mathrm{j}0) - (0.98 - j0.06)]$

$I_{12} = 1.9 - \mathrm{j}0.8$

$I_{21} = -I_{12}$

$I_{13} = y_{13}(V_1 - V_3) = (10 - \mathrm{j}30)[(1.05 + \mathrm{j}0) - (1.0 - j0.05)]$

$I_{13} = 2.0 - \mathrm{j}1.0$

$I_{31} = -I_{13} = -2.0 + \mathrm{j}1.0$

$I_{23} = y_{23}(V_2 - V_3) = (16 - \mathrm{j}32)[(0.98 - \mathrm{j}0.06) - (1.0 - j0.05)]$

$I_{23} = -0.64 + \mathrm{j}4.8$

$I_{32} = -I_{23} = 0.64 - \mathrm{j}4.8$

The line flows are

$S_{12} = V_1 I_{12}^* = (1.05 + j0.0)(1.9 + j0.8) = 1.995 + j0.84\, pu$

$= 199.5 \text{ MW} + \mathrm{j}84.0 \text{ Mvar}$

$S_{21} = V_2 I_{21}^* = (0.98 - j0.6)(-1.9 - j0.8) = -1.91 - j0.67\, pu$

$= -191.0 \text{ MW} - \mathrm{j}67.0 \text{ Mvar}$

$S_{13} = V_1 I_{13}^* = (1.05 + j0.0)(2.0 + j1.0) = 2.1 + j1.05\, pu$

$= 210 \text{ MW} + \mathrm{j}105.0 \text{ Mvar}$

$S_{31} = V_3 I_{31}^* = (1.0 + j0.5)(-2.0 - j1.0) = -2.05 - j0.90\, pu$

$= -205.0 \text{ MW} - j90.0 \text{ Mvar}$

$S_{23} = V_2 I_{23}^* = (0.98 - j0.06)(-0.656 + 0.48) = -0.656 - j0.432\, pu$

$= -65.6 \text{ MW} - j43.2 \text{ Mvar}$

$S_{32} = V_3 I_{32}^* = (1.0 - j0.05)(0.64 + 0.48) = 0.664 + j0.448\, pu$

$= 66.4 \text{ MW} + \mathrm{j}44.8 \text{ Mvar}$

and the line losses are

$$S_{Los.12} = S_{12} + S_{21} = 8.5 \text{ MW} + j17.0 \text{ Mvar}$$
$$S_{Los.13} = S_{13} + S_{31} = 5.0 \text{ MW} + j15.0 \text{ Mvar}$$
$$S_{Los.23} = S_{23} + S_{32} = 0.8 \text{ MW} + j1.60 \text{ Mvar}$$

The power flow diagram is shown in Figure 1-6, where real power direction is indicated by → and the reactive power direction is indicated by ⟶+⟶. The values within parentheses are the real and reactive losses in line.

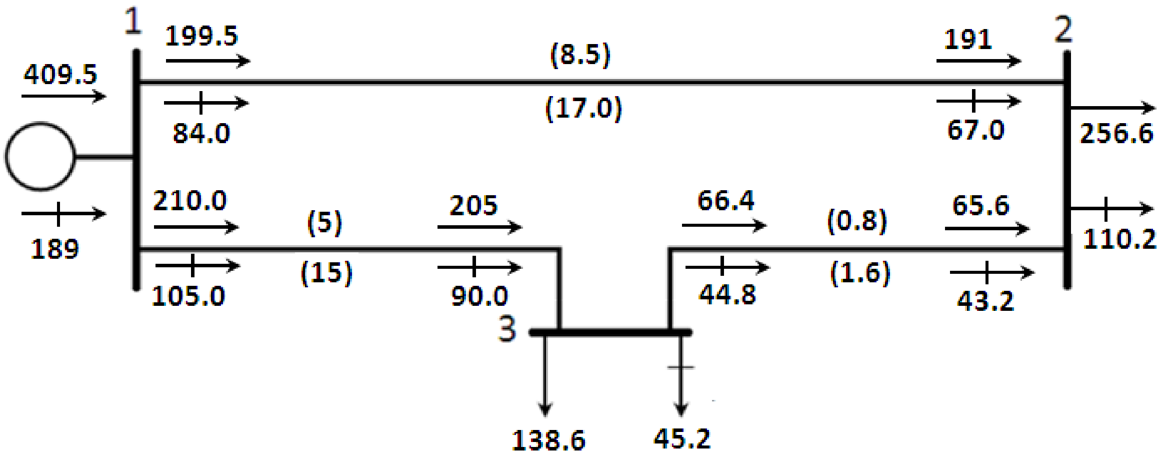

Figure (1-11) Power flow of Example 1-7 **(powers in MW and MVAr)**

Example 1-8

Figure 1-12 shows the one-line diagram of simple three-bus power system with generation at bus 1 and bus 2. Voltage magnitude at bus 3 is fixed at 1.04 pu with a real power generation of 200 MW. A load consisting of 400 MW and 250 MVAr is taken from bus 2. Line impedances marked in per unit on a 100 MVA base, and the line charging susceptances are neglected. Obtain the power flow solution by the Gauss-Seidel method including line flows and line losses.

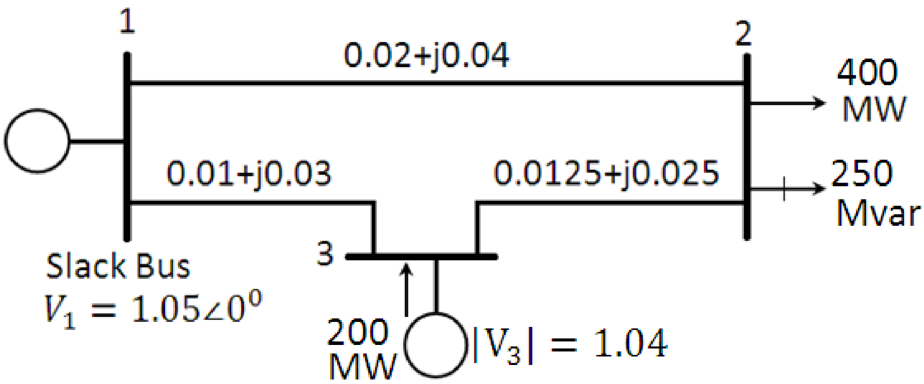

Figure (1-12) One-line diagram of Example 1-8 (impedances in pu on 100 MVA base)

(a) Line impedances are converted to admittances

$$y_{12} = \frac{1}{0.02 + j0.04} = 10 - j20$$

Similarly, $y_{13} = 10 - j30$, $y_{23} = 16 - j32$. The admittances are on the network shown in Figure 1-12b.

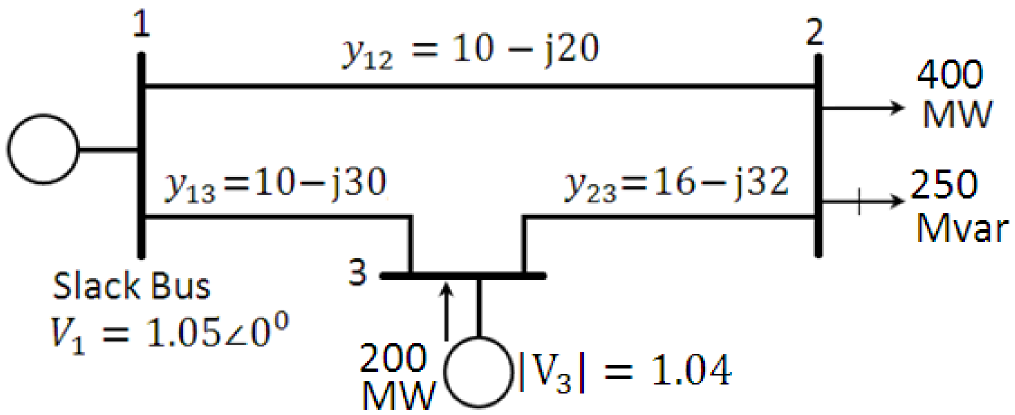

Figure (1-12b) One-line diagram of Example 1-7 **(admittances in pu on 100 MVA base)**

At the P-Q buses, the complex lodes expressed in per units are

$$S_2^{sch} = -\frac{(400 + j250)}{100} = -4.0 - j2.5\ pu$$

$$P_3^{sch} = -\frac{200}{100} = 2.0\ pu$$

Bus 1 is taken as reference bus (slack bus). Starting from an initial estimate of $V_2^{(0)} = 1.0 + j0.0\ and\ \ V_3^{(0)} = 1.04 + j0.0,$

$V_2\ and\ \ V_2$ are computed from (1-28)

$$V_i^{(k+1)} = \frac{\frac{P_i - jQ_i}{V_i^{*(k)}} + \sum_{j=0}^{N} y_{ij} V_j^k}{\sum_{j=0}^{N} y_{ij}} \quad j \neq i$$	(1-28)

$$V_2^{(1)} = \frac{\frac{P_2^{sch} - jQ_2^{sch}}{V_2^{*(0)}} + y_{12}V_1 + y_{23}V_3^{(0)}}{y_{12} + y_{23}}$$

$$V_2^{(1)} = \frac{\frac{-4.0 + j2.5}{1.0 - j0.0} + (10 - j20)(1.05 + j0.0) + (16 - j32)(1.04 + \mathrm{j}0.0)}{(26 - \mathrm{j}52)}$$

$V_2^{(1)}$=0.97462-j0.042307

Bus 3 is a regulated bus where voltage magnitude and real power are specified. For the voltage-controlled bus, first the reactive power is computed from (1-30)

$P_i^{(k+1)} = Re\{V_i^{*(k)}[V_i^{(k)}\sum_{j=0}^{N} y_{ij} - \sum_{j=1}^{N} y_{ij}V_j^{(k)}]\} \quad j \neq i$	(1-29)
$Q_i^{(k+1)} = -Im\{V_i^{*(k)}[V_i^{(k)}\sum_{j=0}^{N} y_{ij} - \sum_{j=1}^{N} y_{ij}V_j^{(k)}]\} \; j \neq i$	(1-30)

$$Q_3^{(1)} = -Im\{V_3^{*(0)}[V_3^{(0)}(y_{13} + y_{23}) - y_{13}V_1 - y_{23}V_2^{(1)}]\}$$

$Q_3^{(1)}$=-I$_m${(1.04-j0)[(1.04+j0)(26-j62)-(10-j30)(1.05+j0)-(16-j32)(0.97462-j0.042307)]}

$$Q_3^{(1)} = 16$$

The value of $Q_3^{(1)}$ is used as Q_3^{sch} for the computation of voltage at bus 3. The complex voltage at bus 3, denoted by $V_{c3}^{(1)}$, is calculated

$$V_{c3}^{(1)} = \frac{\frac{P_3^{sch} - jQ_3^{sch}}{V_3^{*(0)}} + y_{13}V_1 + y_{23}V_2^{(1)}}{y_{13} + y_{23}}$$

$$V_{c3}^{(1)} = \frac{\frac{2.0 - j1.16}{1.04 - j0.0} + (10 - \mathrm{j}30)(1.05 + \mathrm{j}0.0) + (16 - \mathrm{j}32)(0.97462 - j0.042307)}{(26 - \mathrm{j}62)}$$

$$V_{c3}^{(1)} = 1.03783 - j0.005170$$

Since $|V_3|$is held constant at 1.04 pu, only the imaginary part of $V_{c3}^{(1)}$is retained, i.e. $f_3^{(1)} = -0.005170$, and its real part is obtained from

$$e_3^{(1)} = \sqrt{(1.04)^2 - (0.005170)^2}=1.039987$$

Thus

$$V_3^{(1)} =1.039987-j0.005170$$

For the <u>second</u> iteration, we have

$$V_2^{(2)} = \frac{\frac{P_2^{sch} - jQ_2^{sch}}{V_2^{*(1)}} + y_{12}V_1 + y_{23}V_3^{(1)}}{y_{12} + y_{23}}$$

$$V_2^{(2)} = \frac{\frac{P_2^{sch} - jQ_2^{sch}}{V_2^{*(1)}} + y_{12}V_1 + y_{23}V_3^{(1)}}{y_{12} + y_{23}}$$

$$V_2^{(2)} = \frac{\frac{-4.0+j2.5}{0.97462-j0.042307} + (10 - j20)(1.05) + (16 - j32)(1.039987 - j0.005170)}{(26 - j52)}$$

$$V_2^{(2)} = 0.971057 - j0.043432$$

$$Q_3^{(2)} = -Im\{V_3^{*(1)}[V_3^{(1)}(y_{13} + y_{23}) - y_{13}V_1 - y_{23}V_2^{(2)}]\}$$

$Q_3^{(2)}$=-I$_m${(1.039987-j0.005170)[(1.039987$-j$0.005170)(26-j62)-(10-j30)(1.05+j0)-(16-j32)($0.971057 - j0.043432$)]}

$$Q_3^{(2)} = 1.38796$$

$$V_{c3}^{(2)} = \frac{\frac{P_3^{sch} - jQ_3^{sch}}{V_3^{*(1)}} + y_{13}V_1 + y_{23}V_2^{(2)}}{y_{13} + y_{23}}$$

$$V_{c3}^{(2)} = \frac{\frac{2.0-j1.38796}{1.039987+j0.005170} + (10 - j30)(1.05) + (16 - j32)(0.971057 - j0.043432)}{(26 - j62)}$$

$$V_{c3}^{(2)} = 1.03908 - j0.00730$$

Since $|V_3|$ is held constant at 1.04 pu, only the imaginary part of $V_{c3}^{(2)}$ is retained, i.e. $f_3^{(2)} = -0.00730$, and its real part is obtained from

$$e_3^{(2)} = \sqrt{(1.04)^2 - (0.00730)^2}=1.039974$$

or

$$V_3^{(2)} = 1.0399974 - j0.00730$$

The process is continued and a solution is converged with an of accuracy of 5$\times 10^{-5}$ per unit in seven iterations as given below.

$V_2^{(3)}$=0.97073-j0.04479, $Q_3^{(3)} = 1.42904 V_3^{(3)}$=1.03996-j0.00833

$V_2^{(4)}$=0.97065-j0.04533, $Q_3^{(4)} = 1.44833 V_3^{(4)}$=1.03996-j0.00873

$V_2^{(5)}$=0.97062-j0.04555, $Q_3^{(5)} = 1.45621 V_3^{(5)}$=1.03996-j0.00893

$V_2^{(6)}$=0.97061-j0.04565, $Q_3^{(6)} = 1.45947 V_3^{(6)}$=1.03996-j0.00900

$V_2^{(7)}$=0.97061-j0.04569, $Q_3^{(7)} = 1.46082 V_3^{(7)}$=1.03996-j0.00903

The final solution is

$$V_2 = 0.97168 \angle -2.6948^0 \text{pu}$$

$$S_3 = 2.0 + j1.4617\ pu$$

$$V_3 = 1.04 \angle -0.498^0\ \ pu$$

with the knowledge of all bus voltages, the slack bus powers is obtained from (1-27)

$\frac{P_i - jQ_i}{V_i^*} = V_i \sum_{j=0}^{N} y_{ij} - \sum_{j=1}^{N} y_{ij} V_j \qquad j \neq i$	(1-27)

$$P_1 - jQ_1 = V_1^* [V_1 (y_{12} + y_{13}) - (y_{12} V_2 + y_{13} V_3)]$$

$$S_1 = 2.1842 + j1.4085\ pu$$

Line flow and line losses are computed and the result expressed in MW and MVAr

$S_{12} = 179.36\ + \text{j}118.734,$ $\qquad S_{21} = -170.97 - \text{j}101.947$

$S_{13} = 39.06 + \text{j}22.118\,,$ $\qquad S_{31} = -38.88 + \text{j}21.569$

$S_{23} = -229.03 - \text{j}148.05\,,$ $\qquad S_{32} = 238.88 + j167.746$

$S_{Los.12} = S_{12} + S_{21} = 8.39 + \text{j}16.79$

$S_{Los.13} = S_{13} + S_{31} = 0.18 + \text{j}0.548$

$S_{Los.23} = S_{23} + S_{32=}9.85 + \text{j}19.69$